

# Top-Charm flavor changing contributions to the effective $bsZ$ vertex

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## Abstract

We analyze the effects of a tree level flavor changing  $tcZ$  vertex induced by a mixing with new isosinglet  $Q = 2/3$  quarks, on the effective  $bsZ$  vertex. We compute the contributions arising from the new electroweak penguin diagrams involving one insertion of the  $tcZ$  vertex. We show that a generalized GIM mechanism ensures the cancellation of the mass independent terms as well as of the new divergences. Unexpectedly, the presence of a  $tcZ$  coupling cannot enhance the rates for the  $Z$  mediated flavor changing decays  $b \rightarrow s \ell^+ \ell^-$  and  $b \rightarrow s \nu \bar{\nu}$ , implying that these processes cannot be used to set limits on the  $tcZ$  coupling. The additional effects of the heavy isosinglets are compared with the well studied effects of new isodoublets appearing in multi-generational models.

In the Standard Model (SM) Flavor Changing Neutral Current (FCNC) processes are strongly suppressed due to the GIM mechanism. The experimental confirmation of this suppression can be regarded as one of the successes of the theory, and at the same time it represents a challenging constraint for most new physics scenarios which often predict new sources of FCNC. In several new physics models, the new sources of FCNC are related to ratios between the masses of the fermions involved in the FC transitions and some new mass scale, of the order of the electroweak breaking scale or larger. This is the case for example in models where FCNC arise from a mixing between the light fermions and new heavy states with non-standard  $SU(2)_L$  assignments [1–4], in multi Higgs doublets models without natural flavor conservation [5,6] or in models which try to explain the fermion mass hierarchy by means of horizontal symmetries [7]. Due to the smallness of the fermion masses, in all these cases the new FCNC effects are naturally suppressed.

However, if this is the underlying mechanism responsible for the observed suppression, then the absence of FCNC at low energies does not imply the same suppression at large mass scales. In particular, due to the large value of  $m_t$  such a suppression might not be effective for FC transitions involving the top quark [5,6]. Recently some attention has been paid to study this kind of  $tc$  FC transitions, both from the point of view of model building [5,6] as well as from the point of view of the possible phenomenological consequences [8–10].

In this letter we investigate the consequences of a tree level  $tcZ$  FCNC vertex arising from a mixing between the known  $u$ -type quarks and *new*  $Q = 2/3$  isosinglet heavy states on the effective  $bsZ$  vertex. In spite of the loop suppression, there are good reasons for carrying out an analysis of these effects. Under general assumptions, the strength of the  $u_i u_j$  FCNC coupling to the  $Z$  boson induced by a mixing is expected to be of order  $\sim m_i m_j / M^2$ , where  $M$  is the mass-scale of the new states [11]. In this case we would expect that the contribution to the  $bsZ$  vertex induced by penguin diagrams with one insertion of the  $tcZ$  vertex, could be even larger than a  $bsZ$  tree level coupling arising from a similar mechanism, that is from a mixing in the  $d$ -quark sector with new  $Q = -1/3$  isosinglets. Namely, we would expect the ratio between the loop induced and the tree level  $bsZ$  vertices to be

$$\frac{\Gamma_{bsZ}^{\text{penguin}}}{\Gamma_{bsZ}^{\text{tree}}} \sim \frac{V_{tb}^* V_{cs}}{(4\pi)^2} \frac{m_t m_c}{m_b m_s} \sim 2.5 \quad (1)$$

implying that the sensitivity of the  $bsZ$  effective vertex to FC mixing effects could be mainly determined by the presence of a  $tcZ$  coupling.

Apart from inducing FC couplings, a mixing with new isosinglets quarks leads also to the non-unitarity of the  $3 \times 3$  Cabibbo-Kobayashi-Maskawa (CKM) matrix in the

same way as the presence of a fourth generation does. Therefore, the effects induced by one additional  $Q = 2/3$  isosinglet in processes like  $B_d^0 - \bar{B}_d^0$  mixing and  $b \rightarrow s\gamma$ , which are essentially related to the non-unitarity of the CKM matrix, are the same as from a fourth generation, and the same constraints apply in both cases. These effects have been recently analyzed in [12]. The effects of a fourth generation on  $b \rightarrow s\gamma$  were previously studied in [13], and an analysis of the constraints on a fourth generation implied by the experimental measurement of this decay [14], which applies for the isosinglet case as well, has been presented in [15]. The result is that after imposing the limits on the CKM matrix deviations from unitarity, the presence of a new  $Q = 2/3$  isodoublet (or isosinglet) quark in the mass range 200-400 GeV is still consistent with the measured rate for  $b \rightarrow s\gamma$  [14]. Limits on the masses of new fermions in additional generations have been derived also from analyses of the precise electroweak data [3,16], by constraining the contribution to the electroweak radiative corrections from the additional doublets. However, the isosinglets are coupled to the  $SU(2)$  gauge bosons only through small mixings with the standard quarks, and therefore these additional constraints do not apply to the isosinglet case.

With regards to the rare FC decays, the isosinglet case differs from a four generation SM due to the possible presence of FCNC couplings that, as we have stressed, can be particularly large for the  $t$  quark. Since  $Z$  exchange does not contribute either to  $B^0 - \bar{B}^0$  mixing or to  $b \rightarrow q\gamma$ , both these processes are not sensitive to these couplings. However, both the FC decays  $b \rightarrow s\ell^+\ell^-$  and  $b \rightarrow s\nu\bar{\nu}$ , which are strongly suppressed in the SM, are sensitive to  $Z$  exchange. In this case the presence of a  $tcZ$  vertex gives rise to new electroweak penguin diagrams which, being proportional to  $V_{tb}^*V_{cs}$ , are not expected to be suppressed by small CKM mixings. The aim of our analysis is to see whether it is possible to bound a FC  $tcZ$  coupling induced by a mixing with isosinglets by searching for these rare decay modes. Our main results are the following:

- The size of the contribution to the effective  $bsZ$  vertex of the new penguin diagrams induced by a  $tcZ$  vertex is bounded to be smaller than the SM result, and interferes destructively with it. Hence the rate for the FC decays is lowered by this effect, and the experimental upper limits on  $b \rightarrow s\ell^+\ell^-$  [17] and on  $b \rightarrow s\nu\bar{\nu}$  [18] do not imply any constraint on a mixing induced  $tcZ$  vertex.
- The additional effects due to the new diagrams involving loops of the heavy singlets can give an enhancement to the decay rates. However, in the limit of large masses, the isosinglet nature of the new states yields only small logarithmic corrections to the known result for new isodoublets with the same masses [19]. Therefore any signature of the presence of additional  $Q = 2/3$  isosinglets in future measurements of

the  $b \rightarrow s \ell^+ \ell^-$  and  $b \rightarrow s \nu \bar{\nu}$  decay modes, if detected, will not be easily distinguished from the contributions of a heavy fourth generation.

We will first briefly present the general formalism for describing the effects of  $u$ -quark mixing with new isosinglets. Then we will generalize the SM computation of the effective  $bsZ$  vertex [19] to the case when fermion mixing induces tree level FCNC couplings in the  $u$ -quark sector. We will show that in this case a generalized GIM mechanism ensures the finiteness of the result with no need of introducing a cut-off by hand. We note that in general a cut-off is still needed in other cases when a  $tcZ$  coupling does not arise at tree level, but is generated as an effective vertex [10]. Finally, we will discuss the phenomenological implications of our results. Our analysis complements, and sometimes parallels, some recent works on  $tc$  FCNC [10] and on the effects of new heavy  $Q = 2/3$  isosinglets on low energy physics [12,11].

We assume the existence of  $N$  new  $Q = 2/3$  isosinglet  $L$ -handed quarks  $U_L^o$ , as can appear in vector-like multiplets  $U_L^o, U_R^o$  and that they are mixed with the known  $u$ -type quarks  $u_L^o, u_R^{o*}$ . The number  $N$  of  $U_L^o$ - $U_R^o$  pairs is irrelevant for our general analysis, and we will leave it unspecified.  $U_R^o$  and  $u_R^o$ , being both color triplet  $Q = 2/3$  isosinglet states, have the same gauge quantum numbers, and then their couplings to the gauge bosons are not affected by mixing. This is not the case for the  $L$ -chirality states. The vector  $\Psi_{uL}^o = (u^o, U^o)_L^T$  of the doublet ( $u^o$ ) and singlet ( $U^o$ ) gauge eigenstates is related to the corresponding vector of the “light” ( $u$ ) and heavy ( $U$ ) mass eigenstates  $\Psi_{uL} = (u, U)_L^T$  through a unitary matrix  $\mathbf{U}$

$$\begin{pmatrix} u^o \\ U^o \end{pmatrix}_L = \mathbf{U} \begin{pmatrix} u \\ U \end{pmatrix}, \quad \mathbf{U} = \begin{pmatrix} A & E \\ F & G \end{pmatrix}. \quad (2)$$

Here  $U = (U_1, U_2, \dots, U_N)^T$ , while  $u$  is the vector containing the up, charm and top quarks. The unitarity of  $\mathbf{U}$  implies

$$A^\dagger A + F^\dagger F = AA^\dagger + EE^\dagger = I_{3 \times 3}, \quad (3)$$

where  $I_{3 \times 3} = \text{diag}(1, 1, 1)$ . We further introduce a unitary matrix  $K$  for the  $L$ -handed  $d$ -type quarks

$$d_L^o = K d_L, \quad KK^\dagger = K^\dagger K = I_{3 \times 3}. \quad (4)$$

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\* Our results hold also when the  $U_L^o$  isosinglets appear in mirror families  $U_L^o, D_L^o, (U^o D^o)_R^T$ . However in this case the analysis is complicated by the possible appearance of induced  $R$ -handed currents leading to the new effective vertex  $b_{RSR}Z$ .

After introducing the  $(3 + N) \times 3$  matrix

$$P = \begin{pmatrix} I_{3 \times 3} \\ 0 \end{pmatrix}, \quad (5)$$

the Charged Current (CC) coupled to the  $W^\pm$  bosons can be written as

$$\frac{1}{2} J_\mu^W = \bar{\Psi}_{uL}^o \gamma_\mu P d_L^o = \bar{\Psi}_{uL} \gamma_\mu \mathbf{U}^\dagger P K d_L. \quad (6)$$

Then for the CC we can define the  $(3 + N) \times 3$  mixing matrix

$$V = \mathbf{U}^\dagger P K = \begin{pmatrix} V_u \\ V_U \end{pmatrix} = \begin{pmatrix} A^\dagger K \\ E^\dagger K \end{pmatrix}. \quad (7)$$

The  $3 \times 3$  Cabibbo Kobayashi Maskawa (CKM) matrix for the light states  $V_u = (A^\dagger K)$  is not unitary. We note however that (3) and (4) imply

$$V^\dagger V = V_u^\dagger V_u + V_U^\dagger V_U = K^\dagger (A A^\dagger + E E^\dagger) K = I_{3 \times 3}. \quad (8)$$

In terms of the mass eigenstates, the Neutral Current (NC) coupled to the  $Z$  boson reads

$$\frac{1}{2} J_Z^\mu = \frac{1}{2} \bar{\Psi}_{uL} \gamma^\mu \mathbf{U}^\dagger P_{T_3} \mathbf{U} \Psi_{uL} - s_W^2 \bar{\Psi}_u \gamma^\mu Q \Psi_u, \quad (9)$$

where  $s_W^2 = \sin^2 \theta_W$  with  $\theta_W$  the weak mixing angle, and

$$P_{T_3} = P \times P^\dagger = \begin{pmatrix} I_{3 \times 3} & 0 \\ 0 & 0 \end{pmatrix} \quad (10)$$

is the projector on the  $L$ -handed  $T_3 = 1/2$  isospin doublet states. We note that in (9) the second term remains flavor diagonal since the matrix of the electric charges is proportional to the identity ( $Q = \frac{2}{3} I$ ). In contrast, for the isospin part of the current the matrix of the isospin charges  $\frac{1}{2} P_{T_3}$  is not proportional to the identity, and therefore the corresponding isospin couplings are FC. For the NC we can define the following  $(3 + N) \times (3 + N)$  mixing matrix

$$\mathcal{U} = \mathbf{U}^\dagger P_{T_3} \mathbf{U} = \begin{pmatrix} A^\dagger A & A^\dagger E \\ E^\dagger A & E^\dagger E \end{pmatrix} \quad (11)$$

which is also not unitary. However from (4) and (7) and from the first equality in (10), it follows that

$$\mathcal{U} = V \times V^\dagger. \quad (12)$$

The matrix of the FCNC couplings  $\mathcal{U}$  satisfies the following interesting properties:

$$\mathcal{U}\mathcal{U}^\dagger = \mathcal{U}^2 = \mathcal{U}; \quad \mathcal{U}V = V. \quad (13)$$

The first equation tells us that the matrix  $\mathcal{U}$  is idempotent. This is not surprising, since from the first equality in (11) it is clear that  $\mathcal{U}$  can be straightforwardly interpreted as the projector operator on the  $L$ -doublets written in the basis of the mass eigenstates. The second equation has a very important implication. Together with (8) it ensures that, in spite of the presence of the FC couplings, all the mass independent terms in the new penguin diagrams, which carry the structure  $V^\dagger \mathcal{U} V$ , cancel off.

The usual SM  $L$ - and  $R$ -handed chiral couplings of the  $u$  quarks are

$$\varepsilon_L = \frac{1}{2} - \frac{2}{3} s_W^2, \quad \varepsilon_R = -\frac{2}{3} s_W^2. \quad (14)$$

However, from (9) we see that the mixing with the new isosinglets modifies the  $L$ -handed  $u$  coupling, and in particular introduces a FC term. It is convenient to write the general  $\Psi_{u_i} \Psi_{u_j} Z$  coupling as

$$\varepsilon_L^{ij} = \frac{1}{2} \mathcal{U}_{ij} - \frac{2}{3} s_W^2 \delta_{ij} = \varepsilon_L \delta_{ij} + \frac{1}{2} (\mathcal{U}_{ij} - \delta_{ij}), \quad i, j = u, c, t, 1..N. \quad (15)$$

In the second equation, the first term corresponds to a trivial extension of the SM to  $3+N$   $L$ -handed doublets with no tree level FCNC. The second term accounts for the fact that the new  $N$  states are isosinglets. The reason for writing the  $L$ -handed coupling as in (15) is twofold. In the first place, we aim to compare the results for the isosinglets case with those for a multi-generation model, for which only the first term is present. Secondly, through (15) the derivation of the effective  $bsZ$  vertex in the presence of the tree level FC couplings can be more easily performed in two steps. The first step traces trivially the SM computation [19] extended to  $3+N$  generations. As a second step, we need just to compute the two additional diagrams depicted in Fig.1 which arise from the second term in (15).

The sum of the one loop penguin diagrams which do not contain any insertion of the FC couplings yields, in the Feynman gauge,

$$\Gamma_{\text{eff}}^0 = \frac{g^3}{(4\pi)^2 c_W} \bar{b}_L \gamma_\mu s_L \sum_j (V_{jb}^* V_{js}) [X(x_j) + Y(x_j)], \quad (16)$$

where  $c_W = \cos \theta_W$  and  $x_j = m_j^2/M_W^2$  with  $m_j$  the mass of the quark running inside the loop, and

$$X(x_j) = -\frac{5}{4} \left[ \frac{1}{x_j - 1} - \frac{x_j^2 \ln x_j}{(x_j - 1)^2} \right], \quad Y(x_j) = \frac{1}{4} \left[ x_j - \frac{2x_j \ln x_j}{x_j - 1} \right]. \quad (17)$$

This is the known SM result as first given in [19]. The reason for introducing two different functions  $X$  and  $Y$  will become clear in the following. In (16) the sum is taken over all the  $Q = 2/3$  quarks which can appear in the loop. Then Eq. (8), which is analogous to the unitarity of the CKM matrix in the SM and implies  $\sum_j V_{jb}^* V_{js} = 0$ , ensures the correct cancellation of the same set of mass-independent terms and divergences as in the SM case. As is well known, the expressions in (16) is not gauge-invariant by itself. In order to achieve a gauge-invariant result, also the box diagram amplitudes have to be taken into account. For the two processes  $b \rightarrow s \bar{l} l$  (with  $l = \nu, \ell^\pm$ ) we are interested in, the box diagram amplitudes have the quark-mixing structure

$$\mathcal{M}_{\bar{l}l}^{\text{Box}} \propto \sum_j (V_{jb}^* V_{js}) W^l(x_j), \quad (18)$$

where in the Feynman gauge, and neglecting the masses of the charged leptons

$$W^\nu(x_j) = 4 W^{\ell^\pm}(x_j) = 2 \left[ \frac{1}{x_j - 1} - \frac{x_j \ln x_j}{(x_j - 1)^2} \right]. \quad (19)$$

Now the sum

$$\sum_j (V_{jb}^* V_{js}) \left[ X(x_j) + Y(x_j) + W^l(x_j) \right], \quad (20)$$

which appears in the full decay amplitude, is a physical, gauge invariant quantity. The experimental limits on  $b \rightarrow s \ell^+ \ell^-$  [17] and  $b \rightarrow s \nu \bar{\nu}$  [18] can then be used to set bounds on the masses and mixings of the possible new  $U$  doublets contributing to (20). However, other processes can be used to constrain the same parameters. For example, via electromagnetic penguins a quantity analogous to (20) enters the expression for the  $bs\gamma$  effective vertex [19], and thus the presence of the new doublets affects also the rate for the radiative  $b$  decay.

In the case we are analyzing here, the new states are isosinglets, and there are new contributions from the FC couplings. The difference from the  $N$  doublets case is accounted for by the second term in (15). This term gives rise to the two additional diagrams depicted in Fig.1, which involve respectively loops of the  $W$  gauge bosons and of the unphysical scalars  $\phi$ . At a first glance, both the new diagrams appear to be logarithmically divergent. However, the diagram involving the unphysical scalars  $\phi$  is finite due to the presence of the  $P_L$  chiral projector (see Fig.1) which reduces the degree of divergence by a factor of 2. After summing over all the  $u$  and  $U$  fermions, also the diagram involving the  $W$  loop is finite. In fact (8) and (13) imply  $\sum_{jk} V_{jb}^* (\mathcal{U}_{jk} - \delta_{jk}) V_{ks} = 0$  and thus all the terms independent of the  $u$ -quark masses (and in particular the poles at  $D=4$ ) cancel. Such a cancellation in the presence of this kind of tree level FC vertices can be well regarded as a generalization of the SM GIM mechanism.

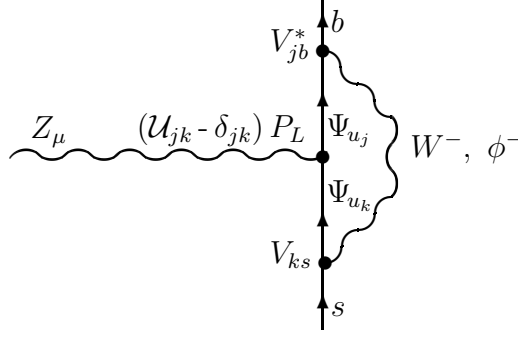


FIG. 1. Electroweak penguin diagrams with  $W$  gauge bosons and unphysical scalars  $\phi$  which include the flavor changing vertex  $\mathcal{U}_{j \neq k}$ . The relevant mixing matrices appearing at the vertices are written explicitly, and  $P_L = \frac{1}{2}(1 - \gamma_5)$  is the  $L$  chiral projector.

The sum of the new  $W$  and  $\phi$  diagrams originating from the second term in (15) which contains the FC vertices reads

$$\Gamma_{\text{eff}}^{FC} = \Gamma_W^{FC} + \Gamma_\phi^{FC} = \frac{g^3}{(4\pi)^2 c_W} \bar{b}_L \gamma_\mu s_L \sum_{j,k} [V_{jb}^* (\mathcal{U}_{jk} - \delta_{jk}) V_{ks}] Z(x_j, x_k), \quad (21)$$

where

$$Z(x_j, x_k) = \frac{1}{4} \frac{1}{x_j - x_k} \left[ \frac{x_k - 1}{x_j - 1} x_j^2 \ln x_j - \frac{x_j - 1}{x_k - 1} x_k^2 \ln x_k \right]. \quad (22)$$

Since no term proportional to  $V_{jb}^* \mathcal{U}_{jk} V_{ks}$  can arise from the box diagrams, beyond being finite (21) is also gauge invariant. Apart from containing a FC part,  $\Gamma_{\text{eff}}^{FC}$  also contains flavor diagonal terms proportional to  $(\mathcal{U}_{jj} - 1)$  corresponding to the limit of equal masses

$$\lim_{x_k \rightarrow x_j} Z(x_j, x_k) = Y(x_j) \quad (23)$$

and  $Y$  is given in (17). By combining (16) and (21) and by means of the limit (23), the effective  $bsZ$  vertex  $\Gamma_{\text{eff}}^Z = \Gamma_{\text{eff}}^0 + \Gamma_{\text{eff}}^{FC}$  can be recast as:

$$\Gamma_{\text{eff}}^Z = \frac{g^3}{(4\pi)^2 c_W} \bar{b}_L \gamma_\mu s_L \sum_j \left[ (V_{jb}^* V_{js}) X(x_j) + (V_{jb}^* \mathcal{U}_{jj} V_{js}) Y(x_j) + \sum_{k \neq j} (V_{jb}^* \mathcal{U}_{jk} V_{ks}) Z(x_j, x_k) \right]. \quad (24)$$

The replacement  $X(x_j) \rightarrow X(x_j) + W^l(x_j)$  in (24) yields a physical quantity directly measurable in  $b \rightarrow s \bar{l} l$  decays. The first term inside the square brackets in (24) is not affected by the fermion mixing. The second term, which is also flavor diagonal, contains a quadratic  $\sim x_j$  dependence which in the SM and in multi-doublet models represents the dominant contribution for very large masses. The mixing with the isosinglets reduces this contribution. In fact, being  $\sum_{j=1}^N \mathcal{U}_{jj} = \text{Tr}(V^\dagger V) = 3$  and since the experimental bounds



on the flavor diagonal  $u_L$  and  $c_L$  mixings [20] imply  $\mathcal{U}_{cc} \sim \mathcal{U}_{uu} \sim 1$ , we have  $\sum_{j=3}^N \mathcal{U}_{jj} \sim 1$ . Hence the dependence on the large masses  $m_t, m_{U_1} \dots m_{U_N}$  is weakened with respect to the doublet case  $\mathcal{U}_{jj} = 1$ . Finally, the third term accounts for the additional effects of the FC vertices. Since for the  $bs\gamma$  effective vertex there are no diagrams analogous to the ones depicted in Fig.1, the rate for  $b \rightarrow s\gamma$  is not sensitive to the FC mixings. Therefore, in the isosinglet case the indirect constraints on  $\Gamma_{\text{eff}}^0$  from the experimental measurement of  $b \rightarrow s\gamma$  [14] cannot be applied to the full  $bsZ$  effective vertex.

In the limit  $|V_{tb}| \sim |V_{cs}| \sim 1$  no additional suppression beyond the loop factor can reduce the effect of a large  $\mathcal{U}_{tc}$ , and then it is interesting to study to what extent such contribution can affect the  $bsZ$  effective vertex. By means of (11) the sum appearing in (21), which accounts for the difference between the isosinglet and isodoublet cases, can be rewritten as a sum over  $j \neq k$  terms involving only the “FC function”  $Z$

$$\begin{aligned} \Gamma_{\text{eff}}^{FC} \propto \sum_{j,k} V_{jb}^* \left( \sum_d V_{jd} V_{kd}^* - \delta_{jk} \right) V_{ks} Z(x_j, x_k) = \\ \sum_{j \neq k} \left[ V_{jb}^* V_{js} (|V_{kb}|^2 + |V_{ks}|^2) + V_{jb}^* V_{jd} V_{kd}^* V_{ks} \right] Z(x_j, x_k). \end{aligned} \quad (25)$$

The second term inside the square brackets can be neglected since it always involves small intergenerational mixings or a small value of  $Z(x_u, x_c)$ , resulting in contributions never exceeding  $10^{-3}$ . As for the first term, it has the standard structure  $V_{jb}^* V_{js}$  and since no ambiguities can arise from phase differences, it can be easily confronted term by term with the standard contributions in (16). Neglecting the possible additional suppression from  $(|V_{cb}|^2 + |V_{cs}|^2)$ ,  $(|V_{cb}|^2 + |V_{cs}|^2) < 1$  the maximum contribution from the FC  $tcZ$  vertex reads

$$(V_{tb}^* V_{ts} + V_{cb}^* V_{cs}) Z(x_t, x_c), \quad (26)$$

which is always smaller in absolute value than the SM term, and of opposite sign. For example, for  $m_t = 180 \text{ GeV}$  and  $m_c = 1.5 \text{ GeV}$  we have  $Z(x_c, x_t) \simeq -0.51$ , while for the leading contribution to the corresponding term in  $\Gamma_{\text{eff}}^0$  we find  $X(x_t) + Y(x_t) + W^{\nu(l^\pm)}(x_t) \simeq 2.59 (2.97)$ . Then the  $tc$  FC contribution to  $\Gamma_{\text{eff}}^Z$  interferes destructively with  $\Gamma_{\text{eff}}^0$  thus *reducing* the  $b \rightarrow sll$  ( $l = \nu, \ell^\pm$ ) decay rates. Therefore we can conclude that it is not possible to translate an upper limit on these decays into a bound on the strength of a  $tcZ$  coupling induced by mixing.

In the scenario we are analyzing here, beyond the effects of the  $tcZ$  coupling there are also other effects related to the presence of the new heavy states, and it is worth studying how these additional effects can influence the effective  $bsZ$  vertex.

The contributions to  $\Gamma_{\text{eff}}^0$  of the heavy isosinglets yield the same enhancement of the effective vertex as for new isodoublets. In fact, a very heavy isosinglet with sizeable

couplings to the  $b$  quark would effectively play the role of a heavier  $t$  quark, thus enhancing  $\Gamma_{\text{eff}}^0$  and the overall effective vertex. However, we stress again that this situation would not affect only the processes involving  $Z$  boson exchange but also, and in a similar way, other processes like  $b \rightarrow s\gamma$ . For the isosinglets, there are specific additional effects from the FC contributions. The effect of a FC mixing of the heavy singlets  $U$  with the  $c$  quark does not differ from the  $tc$  case. Since we always have  $Z(x_U, x_c) < 0$  and  $|Z(x_U, x_c)| < X(x_U) + Y(x_U) + W^{\nu(l^\pm)}(x_U)$  also these terms interfere destructively with the corresponding terms in  $\Gamma_{\text{eff}}^0$ , thus weakening the strength of the effective  $bsZ$  vertex with respect to the doublet case.

If both  $m_i$  and  $m_j$  are  $\gtrsim 150$  GeV, then the function  $Z(x_j, x_k)$  is positive. Therefore the contribution of a  $Ut$  coupling as well as of a pair of new heavy states  $U_1 U_2$  adds constructively to  $\Gamma_{\text{eff}}^0$ . However, in the limit  $x_j \gg x_k \gg 1$  we have  $Z(x_j, x_k) \sim x_k \log x_j$  to be contrasted with the quadratic enhancement  $Y(x_j) \sim x_j$  appearing in  $\Gamma_{\text{eff}}^0$ . Namely, even for large masses, the new FC couplings which are peculiar of the isosinglet case can induce only small positive logarithmic deviations from the isodoublet case. For example, for  $m_U = 500$  GeV and  $m_t = 180$  GeV the presence of a  $\mathcal{U}_{Ut}$  term can enhance the isosinglet case at most by 10 % with respect to the contributions of a new doublet of equal mass.

In conclusion, we have shown that the presence of a mixing induced  $tcZ$  vertex is expected to *lower* the rate for the decays  $b \rightarrow s\ell^+\ell^-$  and  $b \rightarrow s\nu\bar{\nu}$ , and therefore it cannot be constrained by the experimental limits on  $b \rightarrow s\ell^+\ell^-$  [17] and  $b \rightarrow s\nu\bar{\nu}$  [18]. More in general, we have found that the presence of  $Q = 2/3$  isosinglet quarks cannot yield any relevant enhancement of the  $bsZ$  vertex with respect to the better known case of additional doublets, as from a fourth generation, which to some extent is constrained by other rare processes as  $b \rightarrow s\gamma$ . This suggests that it is very unlikely that the peculiar effects of a mixing with isosinglets will be observed in low energy processes as  $b \rightarrow s\ell^+\ell^-$  and  $b \rightarrow s\nu\bar{\nu}$  with the precision in the foreseeable future.

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